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Manuscript received February 4, 1958; revision received July 3, 1958; paper accepted July 7, 1958. Paper presented at A.I.Ch.E. Chicago meeting.

Characteristics of Transition Flow Between Parallel Plates

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Measurements of pressure drop and mean local fluid velocities have been made in a smooth rectangular duct of large aspect ratio. Data have been taken on the steady, isothermal flow of water at room temperature in the viscous, transition, and lower turbulent ranges of flow. Impact probes were installed in the center of the stream, where flow between infinitely broad parallel plates was closely approximated. The limits of the transition range are discussed, and mean local fluid velocities are correlated. Comparison is made with transitional behavior in smooth tubes.

The present investigation was undertaken to gain information about the characteristics of steady, isothermal, fluid flow between smooth, parallel, flat plates. Attention was centered on the region of laminar-turbulent transition, although data were obtained in the viscous and lower turbulent ranges as well. The test fluid was water at room temperature flowing at Reynolds numbers between 865 and 40,190. Measurements of temporal mean local fluid velocities were made by means of a calibrated impact probe, and these data were supplemented by corresponding measurements of the pressure drop caused by fluid friction. Forty-six velocity profiles were obtained to furnish a comprehensive picture of their dependence on the Reynolds number.

Since it was impossible to deal with infinitely broad plates, these were approximated by the long sides of a 0.70- by 14-in. rectangular conduit, 20 ft. in horizontal length, formed from brass plates and equipped with a bell-shaped entrance. The 20:1 aspect ratio of the duct permitted the experimental data to be compared with published information in the fully turbulent range.

Several investigations, such as those of Laufer (*3*), Skinner (*10*), and Wattendorf and Kuethe (*13*), have been made to determine the velocity distribution for air flowing in fully turbulent motion between parallel plates. More recently Sage and coworkers (*2*, *4*) have obtained several excellent velocity profiles in the lower turbulent region. Their apparatus was much like that used in the present experiments. Schlinger and Sage (*8*) have presented a correlation of available velocity data for parallel plates. Using the date of Sage and coworkers as their basis, Rothfus and Monrad (*6*) have developed a means of correlating parallel-plate velocity profiles with those obtained in smooth tubes under fully turbulent conditions. Rothfus and coworkers (*5*) have presented simplified methods of calculating velocity profiles and pressure drops based on the Rothfus and Monrad correlation. The fully turbulent range of flow can therefore be handled reasonably well with respect to fluid friction and the distribution of mean local velocities, provided the calming length is sufficient to make entrance effects negligible.

In the fully viscous range of flow the Navier-Stokes equations of motion can

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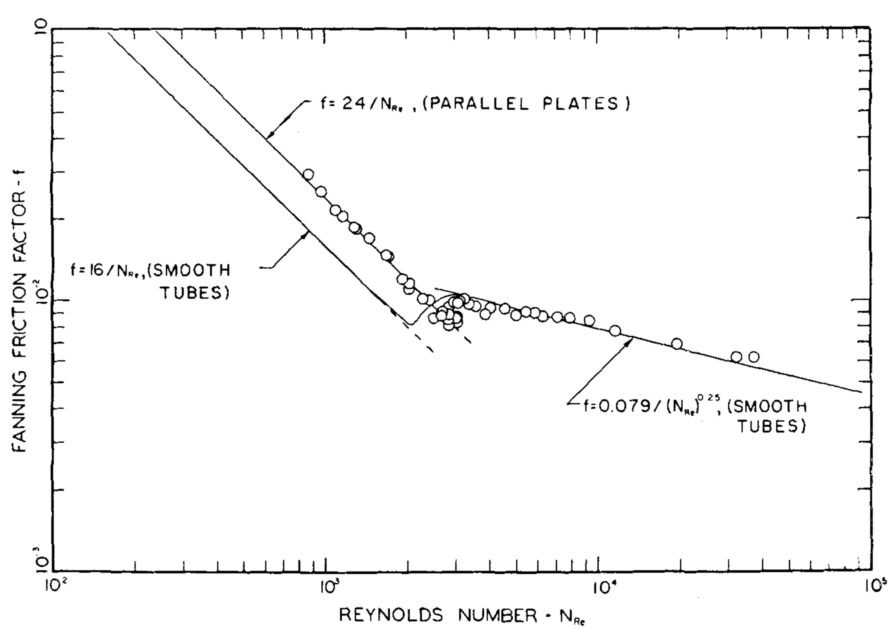

Fig. 1. Fanning friction factors for smooth parallel plates; comparison with smooth-tube correlation is shown.

be integrated without difficulty to obtain the shearing-stress distribution and velocity distribution as well as the pressure loss due to friction (*7a*). The well-known results of this procedure are summarized in the following equations:

Local shearing stress

$$\tau g_c = \frac{\Delta p g_c}{L}(b - y) \quad (1)$$

Local fluid velocity

$$u = \frac{\Delta p g_c}{2\mu L}(2by - y^2) \quad (2)$$

Frictional pressure drop

$$\Delta p = \frac{3\mu V L}{g_c b^2} \quad (3)$$

These equations are based on the assumptions of steady, fully developed viscous motion, zero fluid velocity at the boundaries, constant density and viscosity, and infinitely broad plates.

When one deals with noncircular ducts, it is customary to use a Reynolds number based on the bulk average linear velocity of the fluid and the hydraulic radius of the conduit. For a rectangular duct having a long side of length $2a$ and a short side of length $2b$ the Reynolds number is

$$N_{Re} = \frac{4(4ab)V\rho}{2(2a + 2b)\mu} = \frac{4bV\rho}{\left(1 + \frac{b}{a}\right)\mu} \quad (4)$$

As the aspect ratio a/b increases without limit, thus approaching the case of infinitely broad parallel plates, the Reynolds number becomes

$$(N_{Re})_F = \frac{4bV\rho}{\mu} \quad (5)$$

Regardless of the prevailing type of flow the pressure drop due to fluid friction can be represented by means of the Fanning equation:

$$\Delta p = \frac{2f\rho V^2 L}{g_c D_e} \quad (6)$$

Comparison of Equations (3), (5), and (6) shows immediately that for the case of entirely viscous flow between broad parallel plates

$$f = 24/(N_{Re})_F \quad (7)$$

On the other hand when such flow is fully turbulent, the method of hydraulic radius predicts that

$$f = 0.079/(N_{Re})_F^{0.25} \quad (8)$$

at Reynolds numbers between the upper end of the transition range and 100,000. The latter equation is based on the Blasius friction-factor expression for smooth tubes (*1*), which correlates tubular data at Reynolds numbers between 4,000 and 100,000.

The conditions of pressure drop and velocity distribution on each side of the transition region can therefore be predicted accurately enough for most purposes. Data taken within the transition range should extrapolate to these limits, and the boundaries of the transition zone should prove to be more or less successfully defined on this basis.

EXPERIMENTAL EQUIPMENT

Generally the equipment provided for the flow of water at room temperature from a large constant-head tank to a centrifugal pump, a surge tank, a bell-shaped entry, a test section, a discharge plenum, and thence back to the supply tank. Part of the discharge from the pump could be recycled directly back to the supply tank to control the flow rate in the test section. The surge tank was protected from pump vibrations by a 1-ft. length of flexible tubing in its feed line. The surge tank and test section were mounted on a heavy angle-iron frame which was cushioned against vibrations from the floor.

The test section was fitted with a smooth tapered entry of sheet brass, constructed of brass plates separated by side blocks; the plates were butt-jointed and braced with machined brass strip and angle iron to preserve exact alignment of their junctions. The test section was 20 ft. long.

Five static pressure taps were situated along the center line of the broad upper side of the test section. The velocity profiles were obtained by means of two impact probes situated on and near the center line of the broad upper side of the duct, a distance of 3 ft. from the discharge end. An aluminum feed mechanism permitted the position of the impact opening to be determined within 0.001 in. The impact probes were formed from stainless steel hypodermic-needle tubing having an 0.058-in. O.D. and an 0.009-in. wall thickness. One of the probes was situated on the center line of the channel and the other 1½ in. off the center line. This permitted verification of the opinion expressed by Sage and his associates that the velocity profile along the broad side of such a duct is essentially flat over the middle 3 in.

The manometers used to measure pressure differences were of the ordinary vertical U-tube type. To magnify the readings monochlorobenzene and monofluorobenzene were used as the manometer fluids, thus yielding multiplications of approximately 10 and 40 respectively.

The temperature of the water flowing in the system was measured at the inlet and outlet ends of the test section by mercury-in-glass thermometers. Since the pumps heated the water slightly, cold tap water was bled into the system from time to time to adjust the temperature. The water temperature was maintained within 0.5°C. of the room temperature to assure essentially isothermal flow.

EXPERIMENTAL PROCEDURE

The test section was aligned with the aid of a transit theodolite and leveled to within 0.02 in. over the entire 20 ft. of test length.

Average linear fluid velocities over the middle portion of the conduit were obtained through integration of the experimental velocity profiles in every case. The position of the opening in the impact probe was verified by the determination of both the point of maximum local velocity and the symmetry of the stream. Most of the reported profiles were measured over the upper half of the duct, but at least one point was checked on the other side of the center line in each run. The impact probes were calibrated by the method of Stanton, Marshall, and Bryant (*11*).

The static pressure differences between successive pairs of taps were obtained, and the pressure gradient over the longest

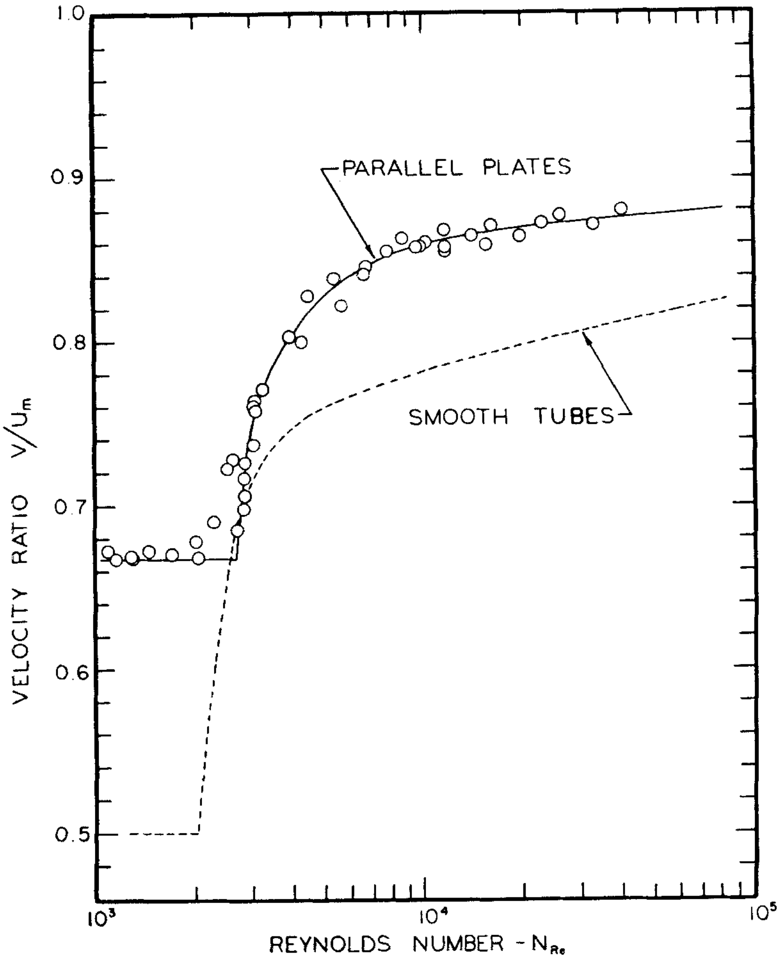

Fig. 2. Effect of Reynolds number on ratio of average to maximum velocity between parallel plates; comparison with smooth-tube correlation is shown.

distance free from entrance effects was used in each case to determine the friction factor.

Velocity measurements were taken after the flow rate had been set and the equipment allowed to run for at least ½ hr. Fifteen to thirty minutes was typically allowed at each point before the pressure differential across the probe was read. After each profile had been obtained, the maximum point was checked, and at least one point on the opposite side of the center line was measured to verify the symmetry of flow. Room and water temperatures were observed and recorded at the time each point velocity was measured.

EXPERIMENTAL RESULTS AND TREATMENT OF DATA

The Fanning friction factors were calculated from the static pressure data by means of Equation (6), with the equivalent diameter taken as four times the half-clearance. The Reynolds number was calculated by means of Equation (5), and the resulting correlation is summarized in Figure 1. For comparison the viscous relationship shown in Equation (7) and the hydraulic-radius prediction shown in Equation (8) are drawn on the same coordinates.

The ratio of average to maximum velocity is shown as a function of Reynolds number in Figure 2. In every case the maximum velocity is a single experimental point, and the bulk average velocity is obtained by means of integration under the measured velocity profile.

The data shown in Figures 1 and 2 can be combined to yield consistent values of the friction velocity parameter at the center of the stream. This is readily apparent from the defining equation, namely

$$u_m^+ = \frac{u_m}{u^*} = \left(\frac{u_m}{V}\right)\left(\frac{1}{f/2}\right) \qquad (9)$$

In addition the center-line value of the friction distance parameter can also be obtained immediately, since by definition

$$y_m^+ = \frac{bu_*\rho}{\mu} = \tfrac{1}{4}N_{ReF}\sqrt{f/2} \qquad (10)$$

The resulting graph of u_m^+ against y_m^+ is shown in Figure 3. Each point represents a combination of experimental V/u_m and friction-factor values.

The nature of the velocity profiles in the transition zone between viscous flow and full turbulence is such that a correlation of the u^+, y^+ type cannot be used conveniently for interpolation. A correlation of the ratio u/u_m as a function of Reynolds number at constant values of the distance ratio y/b however can be interpolated readily. The experimental velocity data were therefore smoothed and correlated as shown in Figure 5. To illustrate the magnitude of agreement between the original data and the final results curves obtained by cross plotting Figure 5 at constant Reynolds numbers are included in Figure 4.

The experimental results summarized in Figures 1, 3, and 5 can be used in the following manner to obtain local velocities

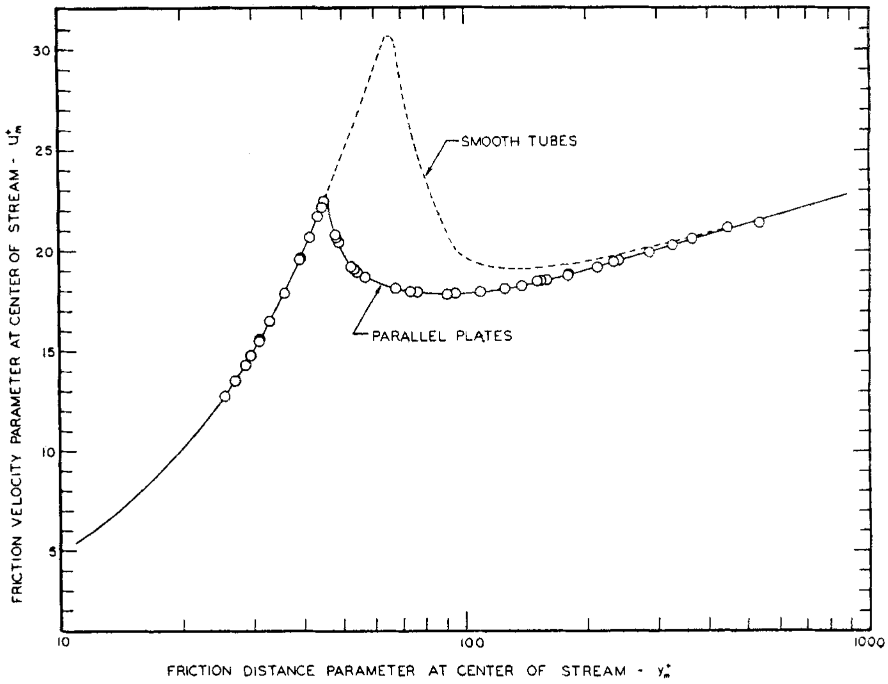

Fig. 3. Relationship between friction-velocity parameter and friction-distance parameter at center of stream flowing between parallel plates; comparison with smooth-tube correlation is shown.

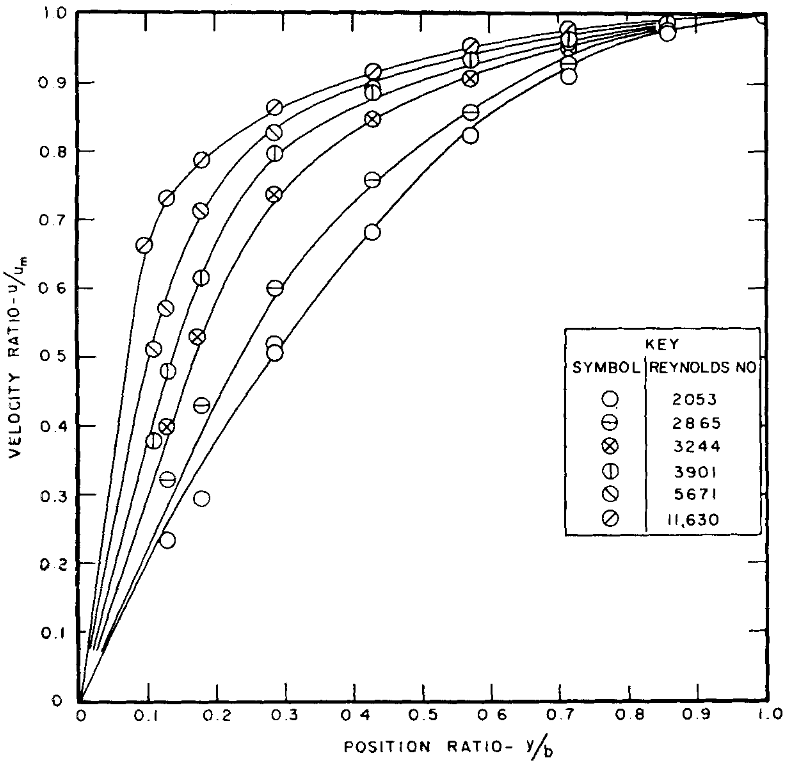

Fig. 4. Velocity profiles for flow between parallel plates. (Solid lines are take from the smoothed correlation shown in Figure 5. Circles show original data.)

at a given Reynolds number. First the friction factor is read from Figure 1. Second the friction velocity is obtained from the equation $u_* = V\sqrt{f/2}$. The parameter u_m^+ is read from Figure 3 at the value of y_m^+ obtained from Equation (10). The center-line velocity is then calculated from the relationship $u_m = u_m^+ u_*$, and the local velocity is obtained from the u/u_m values in Figure 5.

DISCUSSION OF RESULTS

The Fanning friction factors show substantial agreement with the theoretical relationship in viscous flow and the hydraulic-radius prediction in full turbulence. Total agreement can hardly be expected, since the test duct is rectangular and unmeasured-edge effects influence the over-all pressure drop to some extent. The data in the fully turbulent range closely reproduce the results of Sage and his associates. The transition range appears to extend from a Reynolds number of 2,700 to about 3,300 with some evidence of persistence as high as 6,000. There is little or no evidence of any departure from the viscous-flow relationship at Reynolds numbers below 2,700. Smooth tubes on the other hand exhibit a small but definite upward divergence from the laminar line in the region between 1,200 and 2,100 Reynolds number. The dip in Figure 1 however has almost the same depth as for smooth tubes, and the minimum point is marked by the same value of the friction factor.

Figure 2 indicates that the center-line velocity data are consistent with the critical Reynolds number shown by the static pressure data in Figure 1. The scattering of points appears to be unrelated to the direction from which the critical Reynolds number is approached. The smooth-tube line in Figure 2 is based on the data of Senecal and Rothfus (*9*).

Rothfus and Monrad (*6*) have shown that coincident velocity profiles occur in smooth tubes and parallel plates when (1) the flow is fully viscous or fully turbulent, (2) the radius of the tube is equal to the half-clearance between the plates, (3) the skin frictions are equal, and (4) the fluid properties are the same in both conduits. Under such restraints $u_F^+ = u_P^+$ and $y_F^+ = y_P^+$. It follows immediately that the same equalities must hold at the center line of the stream; therefore graphs of u_m^+ against y_m^+ for each configuration should be coincident when drawn on the same set of coordinates.

The parallel-plate data of Figure 3 have been supplemented with a line for smooth tubes which is based for the most part on the experimental results of Senecal and Rothfus (*9*). It is apparent that the two curves are coincident at y_m^+ values less than 45. The latter value represents the critical Reynolds number of 2,700 for parallel plates by virtue of Equaton (10). It is also apparent that the curves merge very gradually at the upper end of the transition zone, and so the point of actual correspondence cannot be stated accurately. At $y_m^+ = 140$, however, the deviation is only about 4%, which is within the combined error of the contributory experiments. This point corresponds to a Reynolds number of 8,800 in the parallel-plate case and a Reynolds number of 4,000 in the smooth tube.

Several conclusions can therefore be reached through examining Figure 3. First the degree of correspondence between the velocity profiles for tubes and parallel plates is a weak function of the Reynolds number in the fully turbulent range. Second the correspondence is greatest at high Reynolds numbers. Third from the practical standpoint the method of Rothfus and Monrad can be

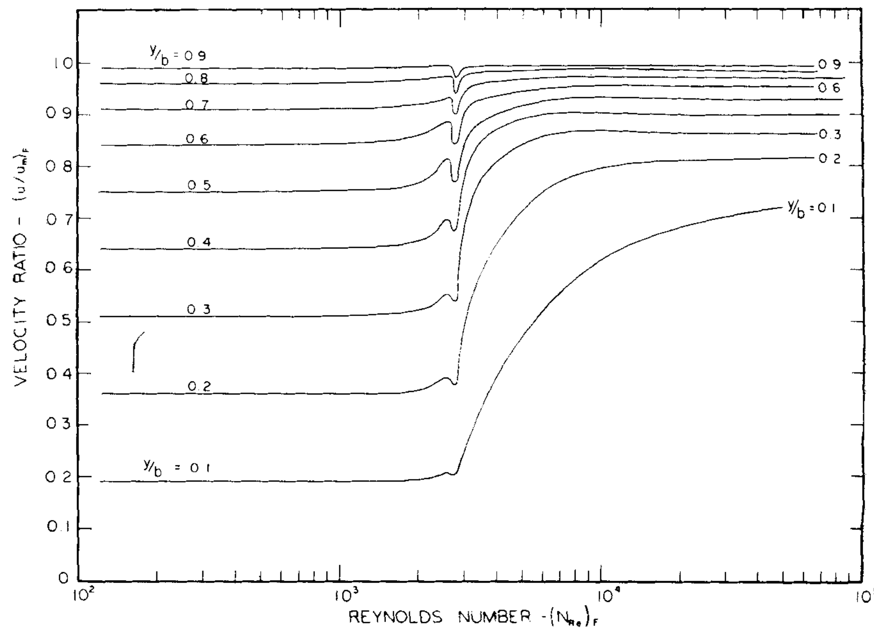

Fig. 5. Correlation of mean local velocities in flow between smooth parallel plates.

used at Reynolds numbers down to the point where the equivalent tube enters transition flow. Fourth the condition of corresponding velocities cannot be used successfully when either conduit is in the transition range. Figure 3 of course deals only with the center line of the stream, but the measured velocity profiles support the stated conclusions.

The behavior of the center-line velocities in the transition range supports the view that local velocities for parallel plates are best correlated in the simple form

$$\frac{u}{u_m} = \phi(N_{Re_F}\ , y/b) \qquad (11)$$

Figure 5 is a picture of this equation which emphasizes the effect of a Reynolds number on the velocity profile while permitting rather easy interpolation among even values of the distance ratio. For comparison the corresponding correlation for smooth tubes has been prepared from the data of Senecal and Rothfus and is shown in Figure 6.

It is apparent that both parallel plates and smooth tubes exhibit much the same behavior in the transition range. The effect of casting off the first large disturbance eddies at the critical Reynolds number is immediately reflected by the velocity profiles in both cases. It should be noted that the velocity ratios at intermediate values of the distance ratio seem to be more greatly disturbed than those near the wall and near the center of the stream. Once the critical point is passed and the frequency of the disturbance eddies increases with increased Reynolds number, the parallel-plate profile approaches its fully turbulent pattern more gradually than does the smooth-tube profile. The velocity pattern therefore supports the frictional data in showing that the transition zone extends over a longer Reynolds-number range in the case of parallel plates than in the case of smooth tubes.

Figures 5 and 6 can be used as working graphs for the calculation of mean local velocities. They should be viewed as purely empirical except in the fully viscous region, although they are also consistent with accepted notions about fully turbulent flow. In the transition zone, which is the local point of the present investigation, there is no adequate basis for treating the data other than empirically at this time.

The relationship between the friction factors shown in Figure 1 and those for concentric annuli of various diameter ratios has been discussed in a recent paper by Walker, Whan, and Rothfus (*12*).

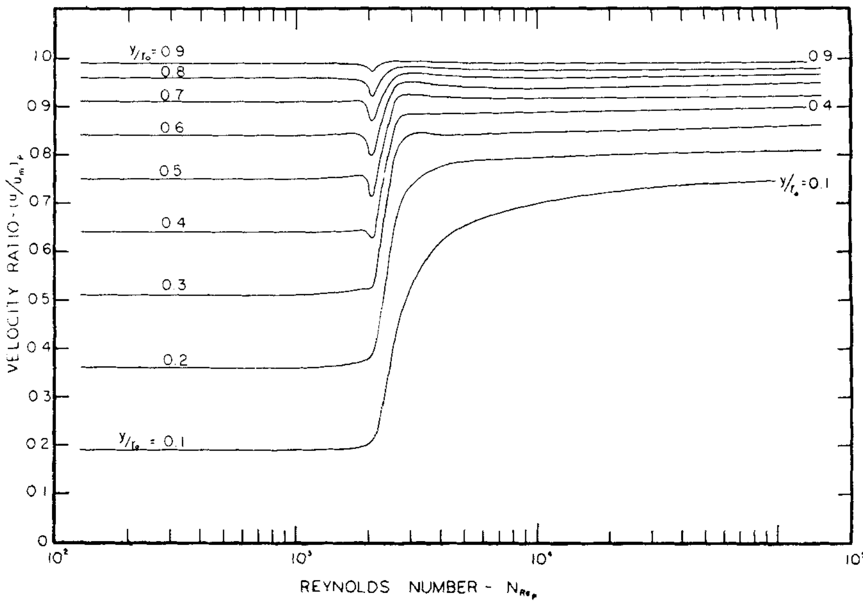

Fig. 6. Correlation of mean local velocities in flow through smooth tubes.

ACKNOWLEDGMENT

This paper was submitted by G. A. Whan in partial fulfillment of the requirements for the degree of Doctor of Philosophy at Carnegie Institute of Technology. Original data and calibrations are presented in the thesis, which is available on interlibrary loan from Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania.

NOTATION

- a = half of long side of rectangular duct, ft.
- b = half of short side of rectangular duct or half-clearance between infinitely broad parallel plates, ft.
- D_e = equivalent diameter of conduit equal to four times the cross-sectional area divided by the wetted perimeter, ft.
- f = Fanning friction factor, dimensionless
- g_c = conversion factor in Newton's second law of motion equal to 32.2 (lb.-mass)(ft.)/(lb.-force)(sec.^{2})
- L = length of conduit over which Δp is measured, ft.
- N_{Re} = Reynolds number equal to $D_e V\rho/\mu$, dimensionless
- Δp = pressure drop due to friction, lb.-force/sq. ft.
- r_0 = radius of tube, ft.
- u = mean local fluid velocity, ft./sec.
- u_* = friction velocity equal to $V\sqrt{f/2}$, ft./sec.
- u^+ = friction velocity parameter equal to u/u_*, dimensionless
- u_m = center-line velocity
- V = bulk average linear fluid velocity, ft./sec.
- y = normal distance from wall to point of measurement in the fluid, ft.
- y^+ = friction-distance parameter equal to $yu_*\rho/\mu$, dimensionless

Greek Letters

- μ = fluid viscosity, lb.-mass/(sec.)(ft.)
- ρ = fluid density, lb.-mass/cu. ft.
- τ = mean local shearing stress, lb.-force/sq. ft.
- ϕ = function

Subscripts:

- F = parallel plates
- m = maximum or center-line value
- P = smooth tubes

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Manuscript received April 4, 1958; revision received August 22, 1958; paper accepted August 26, 1958.